Ampere's loop law

(and a few applications)

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Note : The notes given in this file is no substitute to the much detailed discussion held in the online/contact classes with active participation of students. It , at best, serves the purpose of ready reference for important concepts/derivations covered in the classes.

Ampere's loop

(and a few applications)

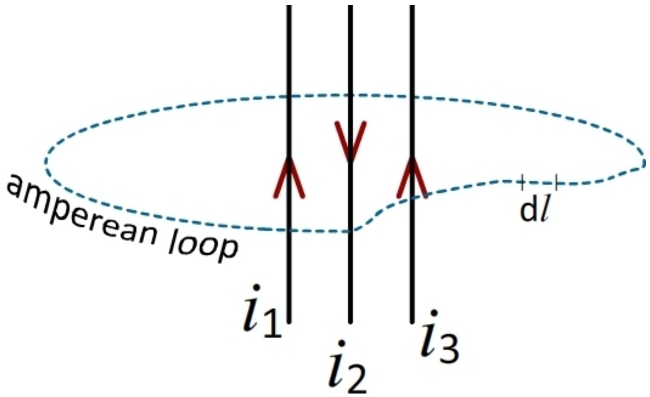
- Ampere's loop law
- $\boldsymbol{B}$ due to an infinite wire
- Force between two parallel conductors
- $\boldsymbol{B}$ due to a solenoid
- $\boldsymbol{B}$ due to a toroid
- $\boldsymbol{B}$ due to a thick wire
- Magnetic dipole moment of an electron

Ampere's loop law

Line integral of a magnetic field around a closed loop is equal to μ_o times the net electric current enclosed by the loop.

$$\oint \overline{B} \bullet d\overline{l} = \mu_o i_{enc}$$

- *Amperean loop* is an imaginary path used to carry out the line integration
- Shape of amperean loop is chosen to simplify the integral
- dl is a small segment of the *amperean loop*
- Ampere's loop law is <u>always applicable</u>, but useful in cases of symmetry (so that computation of integral is convenient)



$$\int \overline{B} \bullet d\overline{l} = \mu_o (i_1 - i_2 + i_3)$$

- Ampere's loop law is NOT the equivalent of Gauss law in magnetism
- Gauss law in magnetism $\oint B \bullet dS = 0$

B due to an infinite straight current carrying conductor

Consider a wire of infinite (nearly) length carrying current i . Let P be a point which is at a perpendicular distance r from the wire. Consider a circular amperean loop concentric with the wire and in a plane perpendicular to the wire.

Magnetic field at any point on the amperean loop is of the same magnitude and along the loop (θ = 0)

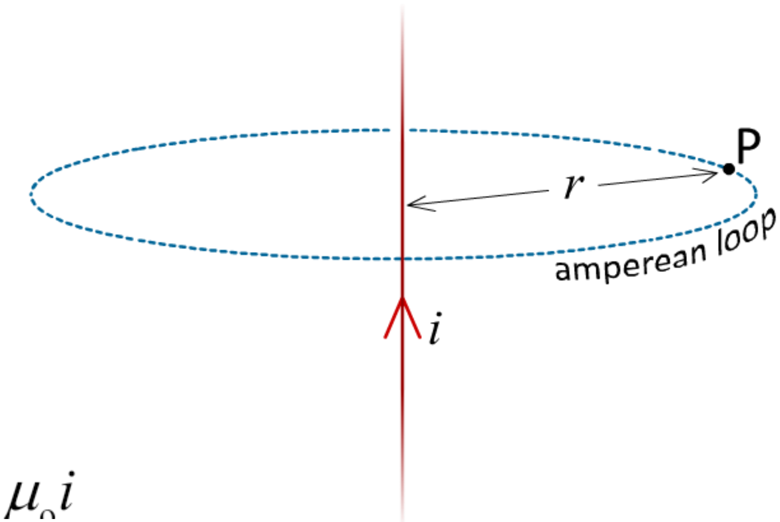
Using ampere's loop law

$$\oint \overline{B} \bullet \mathrm{d}\overline{l} = \mu_o i_{\mathrm{enc}}$$

$$\int B \, \mathrm{d}l \cos(\theta) = \mu_o i_{\mathrm{enc}}$$

$$\int B \, \mathrm{d}l = \mu_o i_{\mathrm{enc}}$$

$$B \int_0^{2\pi r} \mathrm{d}l = \mu_o i$$



$$B \, 2\pi r = \mu_o i$$

$$\boxed{B = \frac{\mu_o i}{2\pi r}}$$

Force between two parallel infinite straight current carrying conductors

Consider two parallel, straight, infinite conductors carrying currents i_1 and i_2 separated by a distance r from each other as shown in the figure

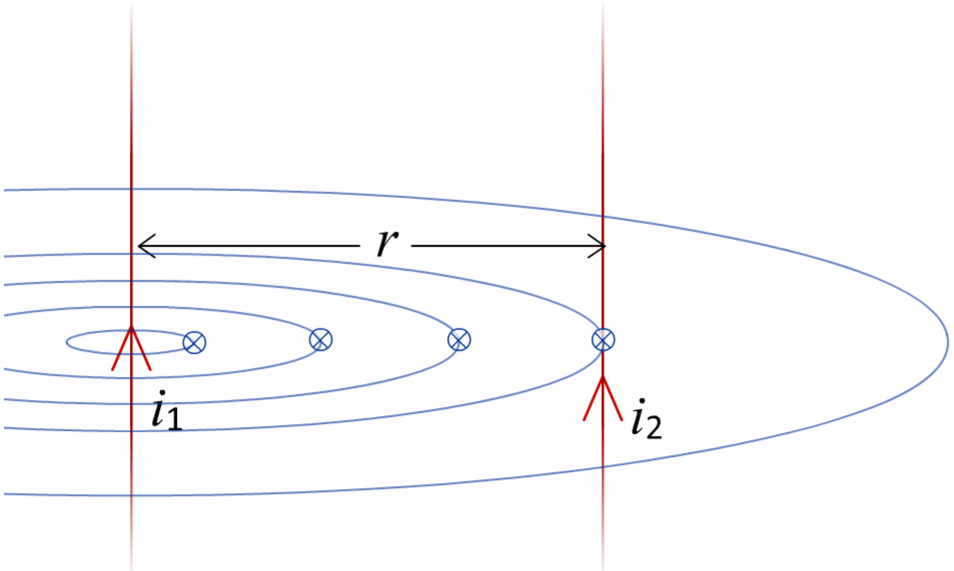
Magnetic field due to the first conductor, at the location of the second conductor, is inwards and is given by the relation

$$B_1 = \frac{\mu_o i_1}{2\pi r}$$

Force experienced by a segment of length l of the second conductor is

$$F = B_1 i_2 l$$

$$F = \frac{\mu_o i_1}{2\pi r} i_2 l$$



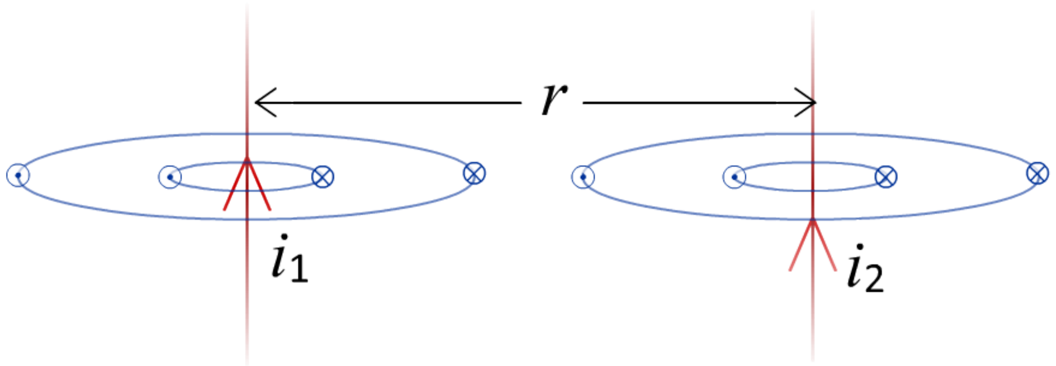
Force per unit length is therefore given by

$$\boxed{\frac{F}{l} = \frac{\mu_o i_1 i_2}{2\pi r}}$$

Two parallel current carrying conductors (further analysis)

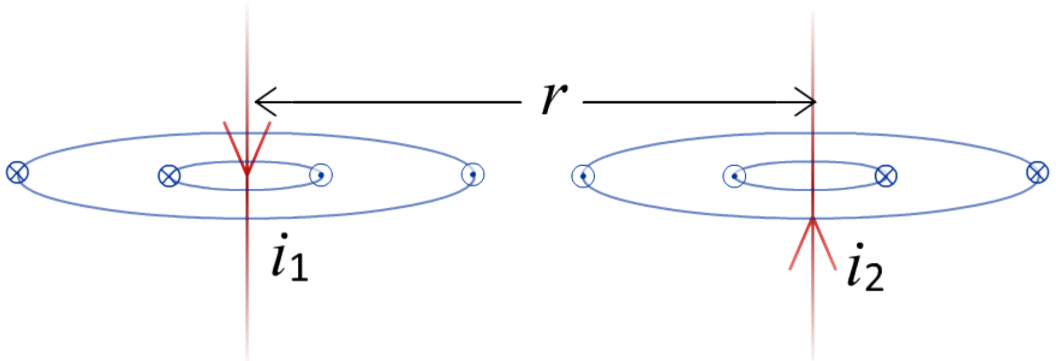
When currents are in the same direction then a null point is obtained at a point between the wires.

Force between the wires is attractive in nature



When currents are in opposite directions, null point is formed beyond the wires

Force between the wires is repulsive in nature



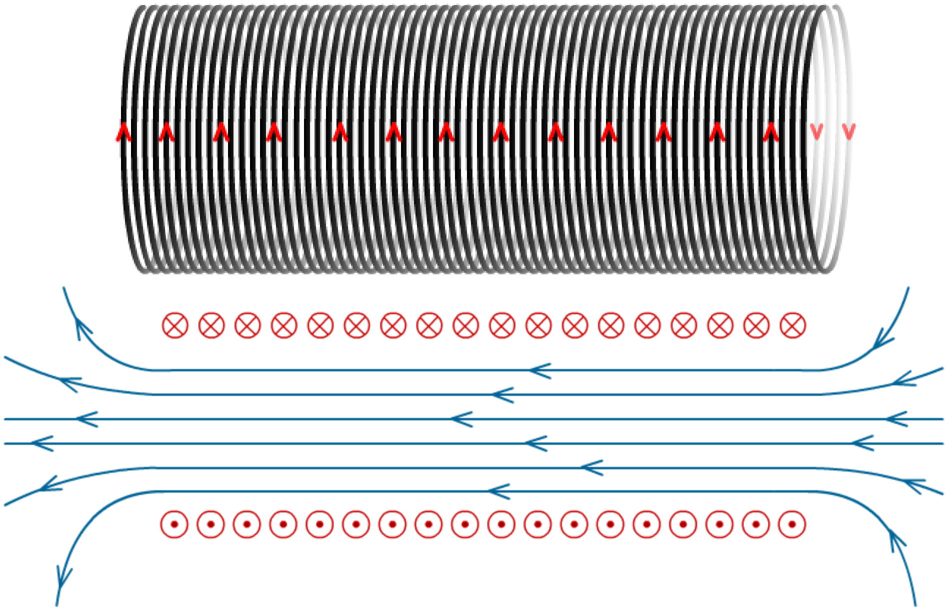
<u>Null point (or neutral point)</u>, in general, is a point where the net magnetic field or electric field or gravitational field is zero.

Magnetic field due to a solenoid

A solenoid is a closely wound helical structure of wire.

When current passes through a solenoid it results in magnetic field.

Magnetic field inside a long solenoid (neglecting fringe effects) may be assumed to be uniform.



Using ampere's loop law

$$\oint \bar{B} \bullet \mathrm{d}\bar{l} = \mu_0 i$$

$$\int B \,\mathrm{d}l \cos(\theta) = \mu_0 N i$$

$$\int B \,\mathrm{d}l = \mu_0 n l i$$

$$B \int \mathrm{d}l = \mu_0 n l i$$

$$B l = \mu_0 n l i$$

$$\boxed{B = \mu_0 n i}$$

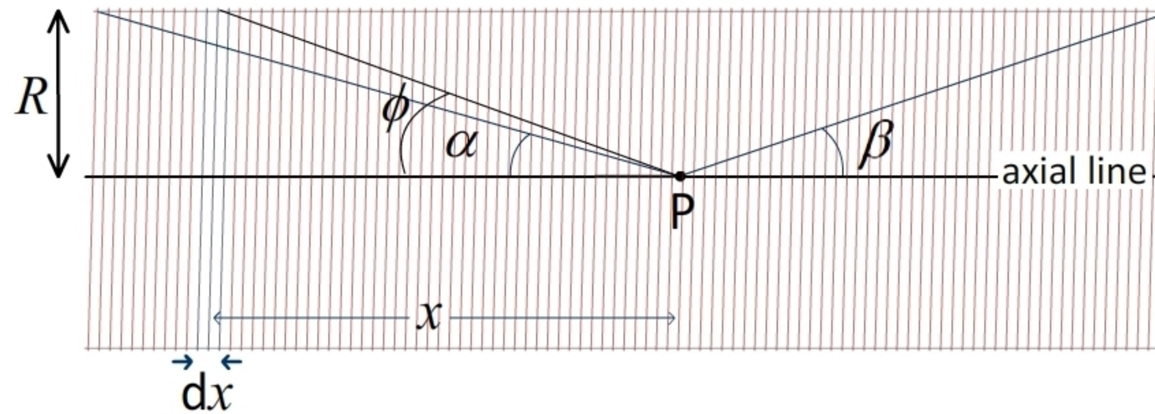
i current in the solenoid

l length of the solenoid

n number of turns per unit length

Magnetic field at a point on the axial line of a finite solenoid (approximation)

Consider a segment of length dl at a distance x , from point P, subtending an angle ϕ w.r.t. the axial line. Magnetic field due to this segment is given by



$$dB = \frac{\mu_o}{4\pi}\frac{2\pi Rn\ i\ dx}{r^2}\sin(\phi)$$

$$dB = \frac{\mu_o}{2}\frac{Rn\ i\ dx}{r^2}\sin(\phi)$$

From the figure $\tan(\phi) = \dfrac{R}{x}$

$$\Rightarrow dx = -\frac{x\sec^2(\phi)}{\tan(\phi)}d\phi$$

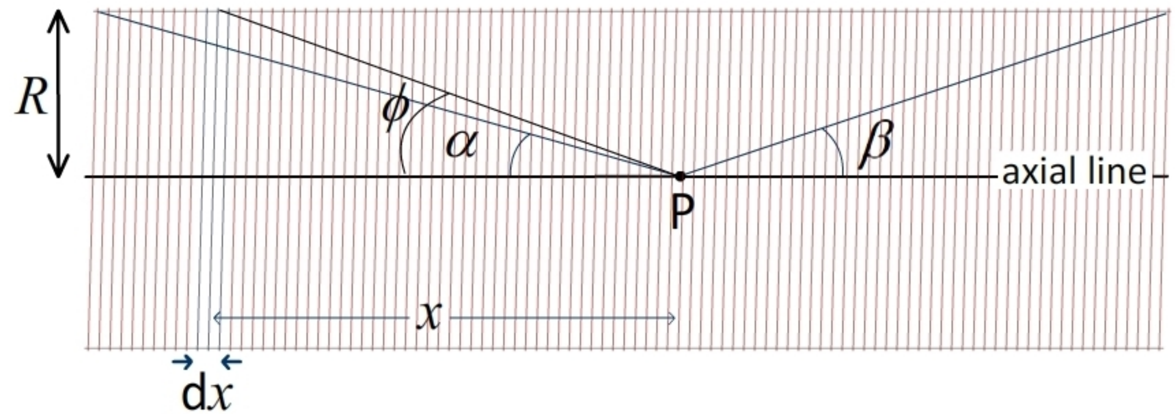
From the figure

$$\sin(\phi) = \frac{R}{r} \quad \Rightarrow \quad r^2 = \frac{R^2}{\sin^2(\phi)}$$

Substituting these we get

$$dB = \frac{\mu_o}{2}\frac{Rni}{R^2}\left[-\frac{x\sec^2(\phi)}{\tan(\phi)}d\phi\right]\sin^3(\phi)$$

Magnetic field at a point on the axial of a finite solenoid (approximation)



$$\mathrm{d}B = \frac{\mu_o}{2}\frac{Rni}{R^2}\left[-\frac{x\sec^2(\phi)}{\tan(\phi)}\mathrm{d}\phi\right]\sin^3(\phi)$$

$$\mathrm{d}B = \frac{\mu_o ni}{2}\left[-\frac{\sec^2(\phi)}{\tan^2(\phi)}\mathrm{d}\phi\right]\sin^3(\phi)$$

$$\mathrm{d}B = -\frac{\mu_o ni}{2}\sin(\phi)\,\mathrm{d}\phi$$

Integrating above expression

$$B = -\frac{\mu_o ni}{2}\int_{-\alpha}^{\beta}\sin(\phi)\,\mathrm{d}\phi$$

$$B = \frac{\mu_o ni}{2}\left[\cos(\phi)\right]_{-\alpha}^{\beta}$$

$$B = \frac{\mu_o ni}{2}\left[\cos(\alpha)+\cos(\beta)\right]$$

For ideal (infinite) solenoid $\alpha = \beta = 0^o$ and the relation reduces to $\mu_o ni$.

Magnetic field due to a toroid

A toroid is a circular solenoid joined end to end. When current passes through a toroid it results in magnetic field confined to a particular region.

Magnetic field inside a tightly wound toroid may be assumed to be uniform.

Using ampere's loop law

$$\oint \overline{B} \bullet d\overline{l} = \mu_o i$$

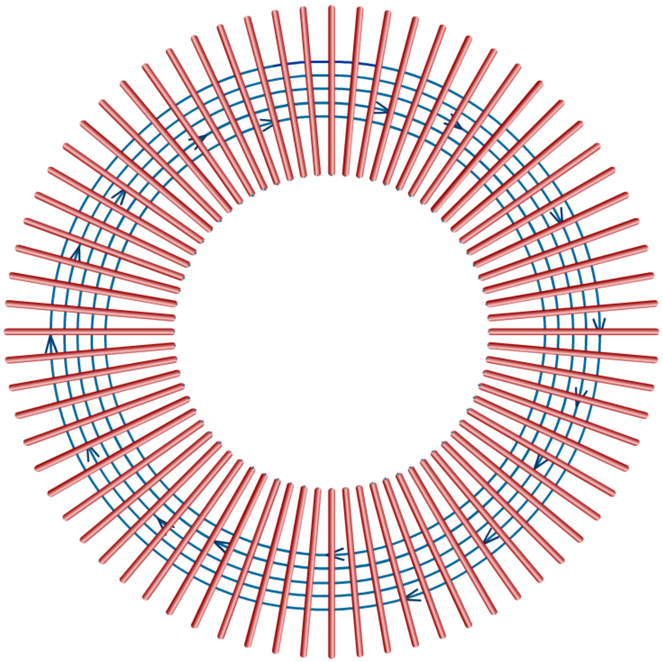
$$\int B \, dl \cos(\theta) = \mu_o N i$$

$$\int B \, dl = \mu_o n 2\pi r i$$

$$B \int dl = \mu_o n 2\pi r i$$

$$B 2\pi r = \mu_o n 2\pi r i$$

$$\boxed{B = \mu_o n i}$$



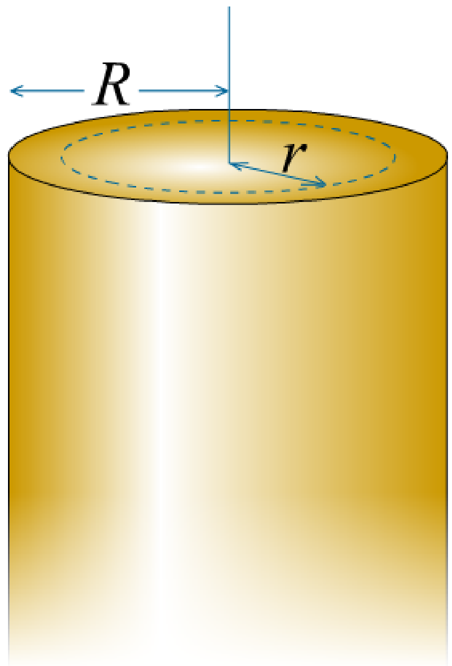
i current in the toroid

r radius of the toroid

n number of turns per unit length

Magnetic field inside and outside a thick wire

Consider a thick wire of radius R. Let J be the uniform current density in the wire.



For a point inside the wire ($r < R$) using ampere's loop law

$$\oint \bar{B} \bullet d\bar{l} = \mu_o i$$

$$\int B\, dl = \mu_o J \pi r^2$$

$$B 2\pi r = \mu_o J \pi r^2$$

$$\boxed{B_{in} = \frac{\mu_o}{2} J r}$$

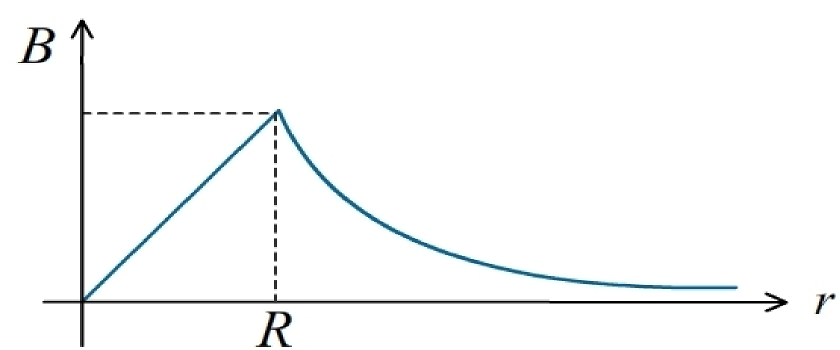
For a point inside the wire ($r \geq R$) using ampere's loop law

$$\oint \bar{B} \bullet d\bar{l} = \mu_o i$$

$$\int B\, dl = \mu_o J \pi R^2$$

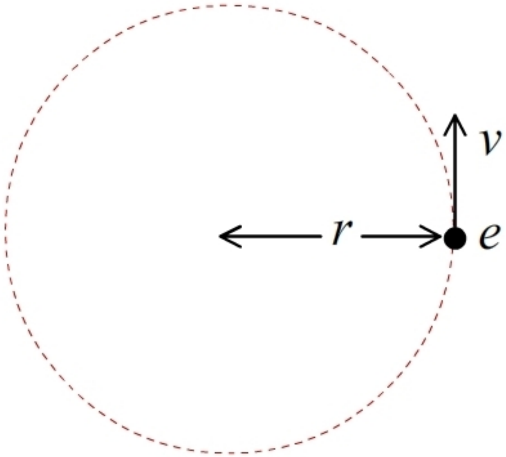
$$B 2\pi r = \mu_o J \pi R^2$$

$$\boxed{B_{out} = \frac{\mu_o J R^2}{2r}}$$



Dipole moment of an electron

Consider an electron of charge e moving in a circular orbit of radius r with uniform speed v.



Effective current due to revolution of electron is given by

$$i = \frac{e}{T}$$

$$i = \frac{e}{2\pi r/v}$$

$$i = \frac{ev}{2\pi r} \quad \text{i}$$

Area (A) enclosed by the circular path is given by

$$A = \pi r^2 \quad \text{ii}$$

Magnetic moment (μ) associated with a current loop is given by

$$\mu = \frac{i}{A}$$

Substituting (i) and (ii) in the above relation we get

$$\boxed{\mu = \frac{evr}{2}}$$